Decision 2 Game Theory Questions

6 Sam is playing a computer game in which he is trying to drive a car in different road conditions. He chooses a car and the computer decides the road conditions. The points scored by Sam are shown in the table.

<div align="center">

Road Conditions

		C_1	C_2	C_3
	S_1	-2	2	4
Sam's Car	S_2	2	4	5
	S_3	5	1	2

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Sam is trying to maximise his total points and the computer is trying to stop him.

(a) Explain why Sam should never choose S_1 and why the computer should not choose C_3.
(2 marks)

(b) Find the play-safe strategies for the reduced 2 by 2 game for Sam and the computer, and hence show that this game does not have a stable solution. *(4 marks)*

(c) Sam uses random numbers to choose S_2 with probability p and S_3 with probability $1 - p$.

    (i) Find expressions for the expected gain for Sam when the computer chooses each of its two remaining strategies. *(3 marks)*

    (ii) Calculate the value of p for Sam to maximise his total points. *(2 marks)*

    (iii) Hence find the expected points gain for Sam. *(1 mark)*

6 Two people, Rowan and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rowan.

Colleen

Strategy	C_1	C_2	C_3
R_1	-3	-4	1
R_2	1	5	-1
R_3	-2	-3	4

Rowan (label for rows)

(a) Explain the meaning of the term 'zero-sum game'. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rowan should never play strategy R_1. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rowan. *(7 marks)*

    (ii) Find the value of the game. *(1 mark)*

4 **(a)** Two people, Ros and Col, play a zero-sum game. The game is represented by the following pay-off matrix for Ros.

		Col		
Strategy		**X**	**Y**	**Z**
	I	-4	-3	0
Ros	**II**	5	-2	2
	III	1	-1	3

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player and state the value of the game. *(2 marks)*

(b) Ros and Col play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros.

		Col		
Strategy		C_1	C_2	C_3
Ros	R_1	3	2	1
	R_2	-2	-1	2

(i) Find the optimal mixed strategy for Ros. *(7 marks)*

(ii) Calculate the value of the game. *(1 mark)*

3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

Callum

		C_1	C_2	C_3
	R_1	5	2	-1
Rose	R_2	-3	-1	5
	R_3	4	1	-2

(a)   (i)   State the play-safe strategy for Rose and give a reason for your answer.

(2 marks)

   (ii)   Show that there is no stable solution for this game.   *(2 marks)*

(b)   Explain why Rose should never play strategy R_3.   *(1 mark)*

(c)   Rose adopts a mixed strategy, choosing R_1 with probability p and R_2 with probability $1 - p$.

   (i)   Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions.   *(3 marks)*

   (ii)   Illustrate graphically these expected gains for $0 \leqslant p \leqslant 1$.   *(2 marks)*

   (iii)   Hence determine the optimal mixed strategy for Rose.   *(3 marks)*

   (iv)   Find the value of the game.   *(1 mark)*